

Coupled eigenmodes in a two-component Bose-Einstein condensate

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Abstract

We have studied the elementary excitations in a two component Bose-Einstein condensate. We concentrate on the breathing modes and find the elementary excitations to possess avoided crossings and regions of coalescing oscillations where both components of the condensates oscillate with the same frequency. For large repulsive interactions between the condensates, their oscillational modes tend to decouple due to decreased overlap. A thorough investigation of the eigenmodes near the avoided crossings are presented.

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I. INTRODUCTION

Bose-Einstein condensation in trapped dilute gases differs from the bulk case in the lack of translational invariance [1–3]. At zero temperature the ensuing spatially varying condensates can be calculated reliably from the Gross-Pitaevskii equations [4]. At finite temperatures the region close to the transition point has been treated by a Hartree-Fock scheme [5,6].

In order to obtain the spectrum of the condensed gases a variety of methods have been applied, most of them utilizing a Bogoliubov-de Gennes diagonalization of the dynamics linearized around the ground state [7–11], and the eigenmodes and their corresponding frequencies are well known [12–16]. The calculations of the temperature dependence of the excitations have still not reached decisive conclusions [17–21].

A new physical situation emerged when it was found that two hyperfine components, $|F = 2, m = 2 \rangle$ and $|F = 1, m = -1 \rangle$, of Rb can be induced to condense concomitantly in a single trap [22]. The two condensates see different potentials, and they tend to appear displaced from each other thanks to the effect of gravity. This is however, a technical feature only and it is possible to make the condensates in a totally symmetric environment [23]. In such a situation, we have shown [24] by a Hartree-Fock treatment that the symmetrically formed condensate may find a state of lower energy by breaking the symmetry. If the experimentally realized condensate falls into the parameter range where this occurs remains to be seen.

In this paper we discuss the oscillational spectrum of a symmetric double condensate. Other works investigating the low lying excitations of the double condensate use a variational approach [25] and a direct numerical treatment [26–28]. We linearize the Gross-Pitaevskii equations of the double condensate and solve for the eigenmodes. Because of the numerical burden of the computations, we only look at the radially symmetric ($l = 0$) oscillations, the breathing modes [12,14]. We have used time dependent integration methods to check the results of our eigenmode analysis, the details will be explained below. As results we have

found the oscillation modes to display avoided crossings as expected of quantum energy levels.

The organization of the present paper is as follows. Section II investigates the small oscillations of the coupled Gross-Pitaevskii equations for the double condensate. Their dynamics is linearized and the usual Bogoliubov-de Gennes expansion [31,32] is generalized to the two-component condensate case, which gives an eigenvalue problem, which is nonhermitian, but has real eigenvalues because of the time reversal symmetry of the problem. In Sec. III we explain how to solve this numerically, and present methods to verify the results by direct time integration of the equations when perturbed from equilibrium. The results of the calculations are presented and discussed in Sec. IV. The main conclusions are presented in Sec. V.

II. FORMULATING THE PROBLEM

We consider a two-component Bose condensed gas in external harmonic potentials

$$V_i(r) = \frac{1}{2}m\Omega_i^2 r^2 \quad i = 1, 2 \quad (1)$$

with the trap frequencies Ω_i and assume a pair potential of the atom-atom interaction of the form $V_i = v_i\delta(\mathbf{r} - \mathbf{r}')$ with

$$v_i = \frac{4\pi\hbar^2 a_i}{m} \quad i = 1, 2, 3 \quad (2)$$

where a_1 and a_2 stand for the inter species scattering lengths and a_3 is the scattering length between the two different atoms, and m is the mass of the atoms which is taken to be the same for both species. The grand canonical Hamiltonian can then be written in the form

$$\begin{aligned} \hat{H} = \int d\mathbf{r} \{ & \hat{\Psi}_1^\dagger(\mathbf{r})[-\frac{\hbar^2}{2m}\nabla^2 + V_1(r) - \mu_1]\hat{\Psi}_1(\mathbf{r}) + \hat{\Psi}_2^\dagger(\mathbf{r})[-\frac{\hbar^2}{2m}\nabla^2 + V_2(r) - \mu_2]\hat{\Psi}_2(\mathbf{r}) \\ & + \frac{1}{2}v_1\hat{\Psi}_1^\dagger(\mathbf{r})\hat{\Psi}_1^\dagger(\mathbf{r})\hat{\Psi}_1(\mathbf{r})\hat{\Psi}_1(\mathbf{r}) + \frac{1}{2}v_2\hat{\Psi}_2^\dagger(\mathbf{r})\hat{\Psi}_2^\dagger(\mathbf{r})\hat{\Psi}_2(\mathbf{r})\hat{\Psi}_2(\mathbf{r}) \\ & + v_3\hat{\Psi}_1^\dagger(\mathbf{r})\hat{\Psi}_2^\dagger(\mathbf{r})\hat{\Psi}_1(\mathbf{r})\hat{\Psi}_2(\mathbf{r}) \} \end{aligned} \quad (3)$$

where $\hat{\Psi}_i(\mathbf{r})$ is the field operator of the bosons and μ_i is the chemical potential. To include this in the Hamiltonian only shifts the energies, and introduces the dependence on the particle number directly into the Hamiltonian.

Using the canonical bosonic commutation relations between the fields $\{\hat{\Psi}_i\}$ we obtain the equations

$$i\hbar \frac{\partial}{\partial t} \hat{\Psi}_1(\mathbf{r}) = [-\frac{\hbar^2}{2m} \nabla^2 + V_1(r) + v_1 \hat{\Psi}_1^\dagger(\mathbf{r}) \hat{\Psi}_1(\mathbf{r}) + v_3 \hat{\Psi}_2^\dagger(\mathbf{r}) \hat{\Psi}_2(\mathbf{r}) - \mu_1] \hat{\Psi}_1(\mathbf{r}) \quad (4)$$

$$i\hbar \frac{\partial}{\partial t} \hat{\Psi}_2(\mathbf{r}) = [-\frac{\hbar^2}{2m} \nabla^2 + V_2(r) + v_2 \hat{\Psi}_2^\dagger(\mathbf{r}) \hat{\Psi}_2(\mathbf{r}) + v_3 \hat{\Psi}_1^\dagger(\mathbf{r}) \hat{\Psi}_1(\mathbf{r}) - \mu_2] \hat{\Psi}_2(\mathbf{r}). \quad (5)$$

Taking the expectation values and assuming the products to factorize, we obtain the time dependent two-component Gross-Pitaevskii equations

$$i\hbar \frac{\partial}{\partial t} \Psi_1(\mathbf{r}) = [-\frac{\hbar^2}{2m} \nabla^2 + V_1(r) + v_1 |\Psi_1(\mathbf{r})|^2 + v_3 |\Psi_2(\mathbf{r})|^2 - \mu_1] \Psi_1(\mathbf{r}) \quad (6a)$$

$$i\hbar \frac{\partial}{\partial t} \Psi_2(\mathbf{r}) = [-\frac{\hbar^2}{2m} \nabla^2 + V_2(r) + v_2 |\Psi_2(\mathbf{r})|^2 + v_3 |\Psi_1(\mathbf{r})|^2 - \mu_2] \Psi_2(\mathbf{r}). \quad (6b)$$

where the chemical potentials are chosen such that

$$\int d\mathbf{r} |\Psi_i(\mathbf{r})|^2 = N_i \quad i = 1, 2. \quad (7)$$

When we set the time derivatives in Eqs. (6a) and (6b) equal to zero, we obtain the Gross-Pitaevskii ground state functions $\{\Psi_1^g, \Psi_2^g\}$ which we assume to be chosen real. They solve the equations

$$\mu_1 \Psi_1^g(\mathbf{r}) = [-\frac{\hbar^2}{2m} \nabla^2 + V_1(r) + v_1 \Psi_1^g(\mathbf{r})^2 + v_3 \Psi_2^g(\mathbf{r})^2] \Psi_1^g(\mathbf{r}) \quad (8a)$$

$$\mu_2 \Psi_2^g(\mathbf{r}) = [-\frac{\hbar^2}{2m} \nabla^2 + V_2(r) + v_2 \Psi_2^g(\mathbf{r})^2 + v_3 \Psi_1^g(\mathbf{r})^2] \Psi_2^g(\mathbf{r}). \quad (8b)$$

As is customary, we now expand the deviation from the ground state in the difference

$$\delta \hat{\Psi}_i(\mathbf{r}) = \hat{\Psi}_i(\mathbf{r}) - \Psi_i^g(\mathbf{r}) \quad (i = 1, 2) \quad (9)$$

which still preserves the operator character of the problem.

We can now proceed in two ways: One way substitutes the expansion (9) into the Hamiltonian (3) and retains the terms to second order only. The linear terms in $\delta \hat{\Psi}_i$ are found

to cancel because Ψ_i^g solves the stationary equations (8a) and (8b). The eigenmodes of the remaining Bogoliubov problem are then obtained by diagonalizing the quadratic part to obtain the diagonal Hamiltonian in term of creation and annihilation operators $\{a_\nu^\dagger, a_\nu\}$ and $\{b_\nu^\dagger, b_\nu\}$, which reduce to the eigenmodes of the separated condensates when the mode coupling v_3 vanishes. In terms of these operators, the Hamiltonian takes the form of decoupled linear oscillator relations.

Alternatively, we may proceed from the equations of motion (6) and linearize these. Finding the eigenmodes, then consists in looking for the eigenfrequencies of this linear time dependent problem. In the present paper we choose to approach the problem in this way.

From the two-component time dependent Gross-Pitaevskii equations (6) we get the linearized equations

$$i\hbar \frac{\partial}{\partial t} \delta\Psi_1 = [\hat{H}_{HO}^{(1)} + 2v_1\Psi_1^{g2}] \delta\Psi_1 + v_1\Psi_1^{g2} \delta\Psi_1^* + v_3(\Psi_2^{g2} \delta\Psi_1 + \Psi_1^g \Psi_2^g \delta\Psi_2^* + \Psi_1^g \Psi_2^g \delta\Psi_2) \quad (10)$$

$$i\hbar \frac{\partial}{\partial t} \delta\Psi_2 = [\hat{H}_{HO}^{(2)} + 2v_2\Psi_2^{g2}] \delta\Psi_2 + v_2\Psi_2^{g2} \delta\Psi_2^* + v_3(\Psi_1^{g2} \delta\Psi_2 + \Psi_1^g \Psi_2^g \delta\Psi_1^* + \Psi_1^g \Psi_2^g \delta\Psi_1) \quad (11)$$

where

$$\hat{H}_{HO}^{(i)} = -\frac{\hbar^2}{2m} \nabla^2 + V_i(\mathbf{r}) - \mu_i. \quad (12)$$

In writing these equations, we have utilized the fact that the ground state functions Ψ_i^g have been chosen real. The time dependent linear equations (10) and (11) can be solved by generalizing the Bogoliubov transformation [32] to the two-condensate inhomogeneous case

$$\delta\Psi_1(\mathbf{r}) = \sum_\nu [\alpha_\nu^{(1)}(\mathbf{r}) \hat{a}_\nu e^{-iE_\nu t/\hbar} + \beta_\nu^{(1)}(\mathbf{r})^* \hat{a}_\nu^\dagger e^{iE_\nu t/\hbar}] \quad (13)$$

$$\delta\Psi_2(\mathbf{r}) = \sum_\nu [\alpha_\nu^{(2)}(\mathbf{r}) \hat{b}_\nu e^{-iE_\nu t/\hbar} + \beta_\nu^{(2)}(\mathbf{r})^* \hat{b}_\nu^\dagger e^{iE_\nu t/\hbar}], \quad (14)$$

where the sum over ν is a sum over elementary excitation modes of the system. Inserting this ansatz into (10) and (11) we find the double condensate Bogoliubov-de Gennes equations

$$[\hat{H}_{HO}^{(1)} + 2v_1\Psi_1^{g2} + v_3\Psi_2^{g2}] \alpha_\nu^{(1)} + v_3\Psi_1^g \Psi_2^g (\alpha_\nu^{(2)} + \beta_\nu^{(2)}) + v_1\Psi_1^{g2} \beta_\nu^{(1)} = E_\nu \alpha_\nu^{(1)} \quad (15a)$$

$$[\hat{H}_{HO}^{(1)} + 2v_1\Psi_1^{g2} + v_3\Psi_2^{g2}] \beta_\nu^{(1)} + v_3\Psi_1^g \Psi_2^g (\alpha_\nu^{(2)} + \beta_\nu^{(2)}) + v_1\Psi_1^{g2} \alpha_\nu^{(1)} = -E_\nu \beta_\nu^{(1)} \quad (15b)$$

$$\left[\hat{H}_{HO}^{(2)} + 2v_2\Psi_2^{g2} + v_3\Psi_1^{g2} \right] \alpha_\nu^{(2)} + v_3\Psi_1^g\Psi_2^g(\alpha_\nu^{(1)} + \beta_\nu^{(1)}) + v_2\Psi_2^{g2}\beta_\nu^{(2)} = E_\nu\alpha_\nu^{(2)} \quad (15c)$$

$$\left[\hat{H}_{HO}^{(2)} + 2v_2\Psi_2^{g2} + v_3\Psi_1^{g2} \right] \beta_\nu^{(2)} + v_3\Psi_1^g\Psi_2^g(\alpha_\nu^{(1)} + \beta_\nu^{(1)}) + v_2\Psi_2^{g2}\alpha_\nu^{(2)} = -E_\nu\beta_\nu^{(2)}. \quad (15d)$$

The ensuing eigenvalue problem is nonhermitian, but it has a symmetry equivalent with that in the case of a single condensate: Taking the complex conjugate, exchanging the α :s and β :s and reversing the sign of the eigenvalue E_ν , we generate equations identical with (15). This time reversal operation guarantees that the eigenvalues are real and come in positive and negative pairs. There is also one pair of eigenvalues which are zero. These eigenvalues correspond to the Gross-Pitaevskii solutions, and are not in this case interesting since they do not represent any oscillations. They correspond to the zero frequency collective modes discussed for the single condensate by Lewenstein and You [33].

III. NUMERICAL METHODS

A. Solving the secular equations

We now consider the case where we have spherically symmetric trapping potentials with $\Omega_1/\Omega_2 = \sqrt{2}$. The coupled Eqs. (8) and (15) are then considerably simplified. In that case, the obvious and simplest way to solve the Bogoliubov-de Gennes equations (15) is to discretize the solutions and the derivatives. This method works well if the problem can be reduced to one dimension. Thus the radial part is solved for, which means that we can only see the so called breathing modes [12,14] of the condensates. When discretizing the solutions $\alpha^{(i)}, \beta^{(i)}$ and the derivatives we get an eigenvalue problem

$$\begin{pmatrix} \hat{M}_1 & v_1\Psi_1^{g2} & v_3\Psi_1^g\Psi_2^g & v_3\Psi_1^g\Psi_2^g \\ -v_1\Psi_1^{g2} & -\hat{M}_1 & -v_3\Psi_1^g\Psi_2^g & -v_3\Psi_1^g\Psi_2^g \\ v_3\Psi_1^g\Psi_2^g & v_3\Psi_1^g\Psi_2^g & \hat{M}_2 & v_2\Psi_2^{g2} \\ -v_3\Psi_1^g\Psi_2^g & -v_3\Psi_1^g\Psi_2^g & -v_2\Psi_2^{g2} & -\hat{M}_2 \end{pmatrix} \begin{pmatrix} \alpha^{(1)} \\ \beta^{(1)} \\ \alpha^{(2)} \\ \beta^{(2)} \end{pmatrix} = E \begin{pmatrix} \alpha^{(1)} \\ \beta^{(1)} \\ \alpha^{(2)} \\ \beta^{(2)} \end{pmatrix} \quad (16)$$

with

$$\hat{M}_i = \hat{H}_{HO}^{(i)} + 2v_i\Psi_i^{g^2} + v_3\Psi_j^{g^2} \quad i \neq j. \quad (17)$$

The eigenvalue problem then concerns a matrix of size $(4n) \times (4n)$, where n is the number of gridpoints used to build up the eigensolutions.

The Bogoliubov-de Gennes equations can also be solved using a basis set method, where the solutions are expanded in some suitable orthonormal set of functions. In the case of harmonic traps, it is favourable to use the eigenstates of the harmonic oscillator. This gives an eigenvalue problem for the expansion coefficients instead of the more direct method of discretizing the solutions. Expanding the solutions in some basis set works particularly well when the condensates are separated by gravity. However, in the effective one-dimensional problem we consider, we found the grid method to be superior. In order to evaluate the matrix in Eq. (16) we need the solutions of the Gross-Pitaevskii equations (8). These were obtained with the method of steepest descent, which has been successfully used in earlier work [29,30] on the nonlinear Schrödinger equation.

B. Time dependent methods

The numerical diagonalization of the secular equation can be checked by a simple time dependent method. Choosing an initial state which is not an eigenstate of Eq. (6a) and (6b) is expected to produce oscillations in the densities. Following the time evolution of these at an arbitrary point in the condensate and fourier transforming the corresponding signal gives us the spectrum of the eigenoscillations. This crude fourier method, where an initial state which slightly differs from the ground state is chosen, works in the spirit of kicking the condensates and letting them ring. It gives us the spectrum in one go but it is not very accurate since we have used only 50 gridpoints in the densities. It is not obvious how to get a good spectrum with this method without greatly increasing the computing work. Increasing the grid to one hundred points has been found to give better agreement with the method in Sec. III.A, but the drawback is that the computing time becomes forty times longer. This method is thus found to be of limited accuracy and has to be considered as a

check on the diagonalization procedure from Eq. (16).

Another approach to calculating the eigenvalue spectrum is to follow the response of the system to an external driving force [31]. This is most easily done directly on the nonlinear equations (6) treated as coupled scalar equations. As discussed in Ref. [31], we add to the external trapping potentials $V_i(r)$ the driving

$$V_d(r) = d \cos(kr + \omega_d t), \quad (18)$$

where ω_d is a variable driving frequency, and k is a suitably chosen scaling parameter, whose value does not greatly affect the results. We can then follow the response of the condensate densities as functions of the driving frequency ω_d by fourier transforming the corresponding changes in the densities as explained above. This method is more accurate and also more transparent, since the condensate response is truly dramatic when ω_d approaches one of the resonance frequencies. One problem is that with a sufficiently strong driving d , harmonic generation is also seen at multiples of the true resonance frequencies. The nonlinearities of the dynamic evolution mixes the frequencies.

IV. RESULTS

The results of our numerical calculations are reported in Fig. 1. The continuous lines are the modes obtained from the secular equation (16) as explained above. Because of the spherical symmetry, the equations are reduced to a one-dimensional eigenvalue problem, which means that we see only the so called breathing modes of the condensates. At $v_3 = 0$ the condensates do not interact with each other and behave like uncoupled systems. On the left hand side we number the eigenfrequencies as #1 to #10 starting from below. At $v_3 = 0$ the frequencies #2, #4, #7 and #9 belong to condensate one, the intertwined ones belong to condensate two. The spectrum is seen to display avoided crossings between #1 and #2 or #6 and #7 around $v_3 = 0.07$.

The discrete points given in Fig.1 derive from the response of the kicked condensate as described in Sec. III.B. As explained there, the results are not very accurate but serve as

an overall check on the spectrum. We found, that choosing a typical initial state to be the Gross-Pitaevskii solution with v_3 slightly shifted ($\Delta v_3 = 0.001$) gives the best spectrum for levels up to $\sharp 4$. With this method it is possible to detect the four lowest frequencies without any major inaccuracy; for higher frequencies the agreement is less satisfactory. The solution of the time dependent Gross-Pitaevskii equations have to be computed for long times in order to get a high resolution spectrum. This was done with the grid method as explained earlier. The results from the fourier method shown in Figs.2-4 used fifty gridpoints in the density. This gives stable iterations and allows us to solve for long times. The frequencies of the spectrum are slightly higher than the spectrum calculated from Eqs. (16). Fig. 5 shows a portion of the spectrum in Fig. 1 when we have increased the number of grid points to 100. We see that the frequencies obtained uniformly tend to approach the results from the secular equations. The fortyfold increase in computer time, however, makes it costly to cover the full spectral range shown in Fig. 1.

The eigenoscillations of the condensates are most clearly observed when the spectrum is investigated using the method of an external driving field. This method produces improved results at least for the three lowest frequencies. In this paper we have used $d = 0.01$ in order to keep $V_d \ll v_i |\Psi_i^g|^2$ and set $k = 1/2$; see Ref. [31]. In Fig. 6 we show the observed response of the two condensates at and near the lowest level with $v_3 = 0.07$. To the left, we see the dramatic increase of the magnitude of the response when ω_d is changed from 3.4 to the resonance value 3.8. As this calculation has been done in the region near the avoided crossing, we find the same frequency nearly equally strong in the fourier transforms of either condensate (to the right in Fig. 6). In Fig. 7, we show the radial densities of condensate one and two. We see that, already for $v_3 > 0.03$, the repulsive interaction tends to separate the condensates thus decreasing their coupling.

In order to illuminate the behaviour of the coupled double condensates, we consider the levels $\sharp 1$ and $\sharp 2$ near $v_3 = 0.07$, where an avoided crossing is suggested. Near $v_3 = 0.0$, level $\sharp 2$ is an excitation in condensate 1 and $\sharp 1$ is condensate 2. If we look at the amplitudes $\alpha_1, \beta_1, \alpha_2$ and β_2 at $v_3 = 0.04$, Fig. 8b shows that the dominating oscillation amplitude

is still α_2 ; the amplitudes α_1 and β_1 in Fig. 8a are clearly smaller. If we compare these amplitudes with the densities of the condensates, Fig. 7, we find that condensate 1 has no oscillational amplitudes after $r \approx 2.5$, because the density goes to zero. The condensate 2 reaches out to about $r \approx 3.5$ because it is repelled by condensate 1. The features in the oscillation amplitudes α_1 and β_1 around $r \approx 2$ are caused by the coupling to condensate 2; for α_2, β_2 the analogous effects are seen near $r \approx 0$. Because the coefficients in Fig. 8 are eigenmodes of the coupled problem, there are areas where the components nearly decouple; thus near $r \approx 2.2$, the oscillations are nearly purely taking place in condensate 1.

When we follow the oscillational level $\sharp 1$ to $v_3 = 0.08$, Fig. 9, we assume that an avoided crossing has been passed. Thus the oscillational amplitudes have been transferred to condensate 1, which is verified in Fig. 9a. The amplitudes in condensate 2, Fig. 9b, are clearly smaller. They disappear rapidly near $r \approx 0$, because for this interaction strength, condensate 2 is pushed well away from the center of the trap, see Fig. 7. Because of the decreased overlap between the condensates, they influence each others oscillation amplitudes far less than for $v_3 = 0.04$.

Table I summarizes the results of all our numerical calculations. The frequencies of the five lowest eigenmodes are reported at $v_3 = 0.002$ and $v_3 = 0.02$. The first column is the result from the Bogoliubov-de Gennes secular equations (15). The next column labelled *Fourier*₁ is obtained from the spectral response of the kicked condensates and the column *Fourier*₂ from the driven condensate response. Comparing the columns we obtain a good picture of the accuracy of our numerical methods. At $v_3 = 0.002$ the frequencies agree well. At $v_3 = 0.02$ the frequencies calculated with the Fourier methods are less satisfactory as compared with the BdG-spectrum. Unfortunately, it is very easy to excite harmonic generations in the vicinity of level $\sharp 4$ and $\sharp 5$, which makes it difficult to find the eigenfrequencies for levels higher than $\sharp 4$. This may also explain the lack of improvement in *Fourier*₂ over *Fourier*₁ at these frequencies.

V. CONCLUSIONS

We have generalized the Bogoliubov-de Gennes method to the case of two coupled condensates. The ensuing eigenvalues are solved for the spherically symmetric excitations, the breathing modes. The calculations are numerically demanding, and obtaining the full spectrum in this way seems to be beyond the numerical capacity of our approach. We have also started from a symmetric ground state, even if we know that the condensates may separate due to symmetry breaking [28]. This we have done mainly for numerical reasons, but our investigations of the condensate stability indicate that the range of interaction we are mainly interested in here, $v_3 \lesssim 0.03$, may lead to stable condensates located on top of each other. We report on these results in a separate communication [34].

We have used time dependent integrations to check the behaviour of the energy eigenvalues in an independent way. Starting from a perturbed state, the system evolution should contain all the eigenfrequencies of the system; here they are restricted to the symmetric ones of course. The advantage of this method is that a fourier transform brings out all frequencies at the same time. This method is found not to be very accurate. However, the method seems to verify the overall behaviour of the modes but in most regions the results fall above those based on the linearization method. This is, however, not true in all parameter ranges, see Fig. 1. A more exact, but quite time consuming method brings the results of the time integration closer to those derived from the linearized method (see Fig 5).

By perturbing the condensates at a single frequency, the resulting response grows dramatically at the resonances (see Fig. 6), but to obtain the full spectrum the calculations have to be repeated for all frequencies of interest. The fourier transform, however, allows one to localize the eigenfrequencies quite accurately (see Table I).

Near the presumed level crossing, $v_3 \approx 0.07$, the two condensates are fully mixed, see Fig. 4. Here, however, the lower frequency, $E \approx 3.8$, is much stronger than the upper one, see Fig. 6. This is clearly seen also at $v_3 = 0.04$ in Fig. 2. The amplitudes of the kicked condensate, however, depend on the point chosen for the fourier transform. For still stronger

couplings $v_3 = 0.08$, the two condensates are only weakly coupled, see Fig. 4. This may be the result of decreasing condensate overlap for large v_3 ; see Fig. 7. The amplitudes of the oscillations in the two condensates, as shown in Fig. 6, should not be directly compared; only the relative strengths of the oscillational components are indicated. In general, we find that level #2 is much harder to excite near $v_3 \approx 0.07$ than level #1 even if both are easily seen in our results. The relative strengths of the two condensate components in one oscillational mode varies with position as the corresponding eigen amplitudes. For level #1, this is shown in Figs. 2,3 and 4. We point out that the region where we have found strong mode mixing, $v_1 \approx v_2 \approx v_3$, does correspond approximately to the physical situation. This suggests that our results may have experimental manifestations in real systems.

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REFERENCES

- [1] M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, *Science* **269**, 198 (1995)
- [2] K.B. Davis, M.-O. Mewes, M.R. Andrews, N.J. van Druten, D.S. Durfee, D.M. Kurn, and W. Ketterle, *Phys. Rev. Lett.*, **75**, 3969 (1995)
- [3] C.C. Bradley, C.A. Sackett, J.J. Tollett, and R.G. Hulet, *Phys. Rev. Lett.* **75**, 1687 (1995)
- [4] M. Edwards and K. Burnett, *Phys. Rev. A* **51**, 1382 (1995)
- [5] D.A.W. Hutchinson, E. Zaremba, and A. Griffin, *Phys. Rev. Lett.* **78**, 1842 (1997)
- [6] P. Öhberg and S. Stenholm, *J. Phys. B* **30**, 2749 (1997)
- [7] S. Stringari, *Phys. Rev. Lett.* **77**, 2360 (1996)
- [8] P. Öhberg, E.L. Surkov, I. Tuttonen, S. Stenholm, M. Wilkens, and G.V. Shlyapnikov, *Phys. Rev. A* **55**, R3346 (1997)
- [9] M. Edwards, P.A. Ruprecht, K. Burnett, R.J. Dodd, and C.W. Clark, *Phys. Rev. Lett.* **77**, 1671 (1996)
- [10] K.G. Singh and D.S. Rokhsar, *Phys. Rev. Lett.* **77**, 1667 (1996)
- [11] L. You, W. Hoston, and M. Lewenstein, *Phys. Rev. A* **55**, R1581 (1997)
- [12] D.S. Jin, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, *Phys. Rev. Lett.* **77**, 420 (1996)
- [13] M.-O. Mewes, M.R. Andrews, N.J. van Druten, D.M. Kurn, D.S. Durfee, C.G. Townsend, and W. Ketterle, *Phys. Rev. Lett.* **77**, 988 (1996)
- [14] M.-O. Mewes, M.R. Andrews, D.M. Kurn, D.S. Durfee, C.G. Townsend, and W. Ketterle, *Phys. Rev. Lett.* **78**, 582 (1997)

- [15] D.S. Jin, M.R. Matthews, J.R. Ensher, C.E. Wieman, and E.A. Cornell, Phys. Rev. Lett. **78**, 764 (1997)
- [16] D.M. Stamper-Kurn, H.-J. Miesner, S. Inouye, M.R. Andrews, and W. Ketterle, Phys. Rev. Lett. **81**, 500 (1998)
- [17] S. Choi, S.A. Morgan, and K. Burnett, quant-ph/9801064
- [18] L.P. Pitaevskii and S. Stringari, cond-mat/9708104
- [19] W.V. Liu, Phys. Rev. Lett. **79** 4056, (1997)
- [20] P.O. Fedichev, G.V. Shlyapnikov, and J.T.M. Walraven, Phys. Rev. Lett. **80**, 2027 (1998)
- [21] S. Giorgini, cond-mat/9709259
- [22] C.J. Myatt, E.A. Burt, R.W. Ghrist, E.A. Cornell, and C.E. Wieman, Phys. Rev. Lett. **78** 586, (1997)
- [23] M.R. Matthews, D.S. Hall, D.S. Jin, J.R. Ensher, C.E. Wieman, E.A. Cornell, F. Dalfovo, C. Minniti, and S. Stringari, Phys. Rev. Lett. **81**, 243 (1998)
- [24] P. Öhberg and S. Stenholm, Phys. Rev. A **57**, 1272 (1998)
- [25] Th. Busch, J.I. Cirac, V.M. Perez-Garcia, and P. Zoller, Phys. Rev. A **56**, 2978 (1997)
- [26] B.D. Esry and C.H. Greene, Phys. Rev. A **57** 1265, (1998)
- [27] H. Pu and M. Bigelow, Phys. Rev. Lett. **80**, 1134 (1998)
- [28] D. Gordon and C.M. Savage, cond-mat/9802247
- [29] F. Dalfovo and S. Stringari, Phys. Rev. A **53**, 2477 (1996)
- [30] B.D. Esry, C.H. Greene, J.P. Burke, and J.L. Bohn, Phys. Rev. Lett. **78**, 3594 (1997)
- [31] P.A. Ruprecht, M. Edwards, K. Burnett, and C.W. Clark, Phys. Rev. A **54**, 4178 (1996)

- [32] A. Fetter, Annals of Physics **70**, 67-101 (1972)
- [33] M. Lewenstein and L. You, Phys. Rev. Let. **77**, 3489 (1996)
- [34] P. Öhberg, *to be published*

FIGURES

FIG. 1. The spectrum of the breathing modes. The lines are the eigenvalues from the Bogoliubov-de Gennes secular equations. The crosses (condensate 2) and the squares (condensate 1) is the spectrum calculated with the fourier method. Throughout the calculations we have used $N_1 = N_2 = 2100, v_1 = 0.02$ and $v_2 = 0.01$. All energies are in units of $\hbar\sqrt{\Omega_1\Omega_2}/2$. The levels are numbered at $v_3 = 0$ from #1 to #10 starting from below. The fourier method enables us to calculate the four lowest modes with a reasonable accuracy, since for higher frequencies the corresponding signal decreases rapidly.

FIG. 2. As an example of our numerical data, this figure shows the spectrum computed by the fourier method at the levels #1 and #2. Here $v_3 = 0.04$ and the two condensates can be seen to simultaneously oscillate with the frequency $E_1 = 3.1$ whereas level #2 is hardly seen at $E_2 = 4.6$. The relative amplitude of the oscillations depends on the point where the density is investigated; cf. Fig. 8.

FIG. 3. The same situation as in Fig.2 with $v_3 = 0.06$. The spectrum shows simultaneous oscillations of both condensates at level #1 with $E_1 = 3.6$ and level #2 with $E_2 = 4.8$.

FIG. 4. Same situation as in Fig.2 and Fig.3 with $v_3 = 0.08$ where the two condensates are decoupled for the two lowest levels.

FIG. 5. A comparison between the Bogoliubov-de Gennes spectrum and the fourier method spectrum calculated with a grid twice as dense as in the previous figures. The general trend with a denser grid is that the values approach those from the secular equation, at the expense of a fortyfold increase in computing time.

FIG. 6. The condensates respond dramatically when ω_d approaches a resonance frequency. The signals in the left part of the figure are the change in the density as function of time for two different driving frequencies. The fourier transform of these signals give the spectra on the right. The time is expressed in units of $2/\sqrt{\Omega_1\Omega_2}$ and E in $1/2\hbar\sqrt{\Omega_1\Omega_2}$. Since the interaction strength is here $v_3 = 0.07$ the responses of both condensates are significant. Oscillations at frequency $\sharp 2$ are hardly seen here.

FIG. 7. The more weakly trapped condensate is forming a shell structure separating the condensates with increasing interaction strength v_3 . Condensate one is concentrated near the center of the trap ($r \approx 0$) and condensate two forms a shell around it.

FIG. 8. The amplitudes of the eigenmodes for level $\sharp 1$ from the Bogoliubov-de Gennes equations shown in a) for $\alpha^{(1)}$ and $\beta^{(1)}$, and in b) for $\alpha^{(2)}$ and $\beta^{(2)}$ at $v_3 = 0.04$.

FIG. 9. The same situation as in Fig. 8 with $v_3 = 0.08$. The roles of the two condensates are changed, which we interpret to derive from passing an avoided crossing.

TABLES

TABLE I. A comparison between the Bogoliubov-de Gennes (BdG) solutions and the fourier methods. Here Fourier₁ stands for the crude method whereas Fourier₂ is the method of driving the condensate with a frequency ω_d . The coalescing oscillations are not shown here.

v_3	BdG	Fourier ₁	Fourier ₂	Level #
0.002	3.43	3.55	3.50	1
	4.99	5.15	5.10	2
	6.62	6.85	6.60	3
	9.34	9.65	9.60	4
	9.88	10.25	9.80	5
0.020	2.85	2.95	3.00	1
	4.59	4.80	4.70	2
	6.17	6.35	6.30	3
	8.54	8.90	9.00	4
	9.63	9.90	10.00	5





